Monday, April 14, 2025

Problem 4. Let ABC be an acute triangle with incentre I and $AB \neq AC$. Let lines BI and CI intersect the circumcircle of ABC at $P \neq B$ and $Q \neq C$, respectively. Consider points R and S such that $AQRB$ and $ACSP$ are parallelograms (with $AQ \parallel RB$, $AB \parallel QR$, $AC \parallel SP$, and $AP \parallel CS$). Let T be the point of intersection of lines RB and SC. Prove that points R, S, T, and I are concyclic.

Problem 5. Let $n > 1$ be an integer. In a *configuration* of an $n \times n$ board, each of the n^2 cells contains an arrow, either pointing up, down, left, or right. Given a starting configuration, Turbo the snail starts in one of the cells of the board and travels from cell to cell. In each move, Turbo moves one square unit in the direction indicated by the arrow in her cell (possibly leaving the board). After each move, the arrows in all of the cells rotate $90°$ counterclockwise. We call a cell *good* if, starting from that cell, Turbo visits each cell of the board exactly once, without leaving the board, and returns to her initial cell at the end. Determine, in terms of n, the maximum number of good cells over all possible starting configurations.

Problem 6. In each cell of a 2025×2025 board, a nonnegative real number is written in such a way that the sum of the numbers in each row is equal to 1, and the sum of the numbers in each column is equal to 1. Define r_i to be the largest value in row i, and let $R = r_1 + r_2 + \cdots + r_{2025}$. Similarly, define c_i to be the largest value in column i, and let $C = c_1 + c_2 + \cdots + c_{2025}$.
What is the largest possible value of $\frac{R}{C}$?